

NAME AND SURNAME :

MATHEMATICS TEACHER :

Hudson Park High School

GRADE 12
MATHEMATICS
June Paper 2

Time : 3 hour

Date : 5 June 2014

Marks : 150

Examiner : SLT

Moderator(s) : SLK and CLM

INSTRUCTIONS

1. Illegible work, in the opinion of the marker, will earn zero marks.
2. Number your answers clearly and accurately, exactly as they appear on the question paper.
3. **NB** Start each question at the top of a page.
4. **NB**
 - Staple your foolscap answers and answer sheets (in the correct order)
 - Fill in the details requested on the front of the question paper and hand your question paper in separately.
5. Employ relevant formulae and show all working out. Answers alone may not be awarded full marks.
6. (Non programmable and non graphical) Calculators may be used, unless their usage is specifically prohibited.
7. Round off answers to 2 decimal places, where necessary, unless instructed otherwise.

QUESTION 1 [7 marks]

1. Given : 10 13 19 21 22 23 26 28 30 30 30 33 34

For this data :

- 1.1.1. Calculate the mean. 1
- 1.1.2. Determine the median. 1
- 1.1.3. Hence, comment on the distribution of the data.
Justify your answer. 2 (4)
- 1.2. Determine the value above which a data value would be classified
as an outlier. (3)

QUESTION 2 [5 marks]

USE THE ANSWER SHEET PROVIDED

2. The relationship between blood alcohol levels and the relative risk of having a car accident was researched. The following table shows the results :

Blood alcohol level (%)	Relative risk of having a car accident (%)
0,00	1
0,05	2,9
0,10	8,5
0,15	24,8
0,20	72,2
0,25	89,5

- 2.1. Draw a scatter plot to represent the data. (2)
- 2.2. Draw in the curve of best fit for the data. (1)
- 2.3. Describe the trend of the data. (2)

QUESTION 3 [10 marks]

USE THE ANSWER SHEET PROVIDED

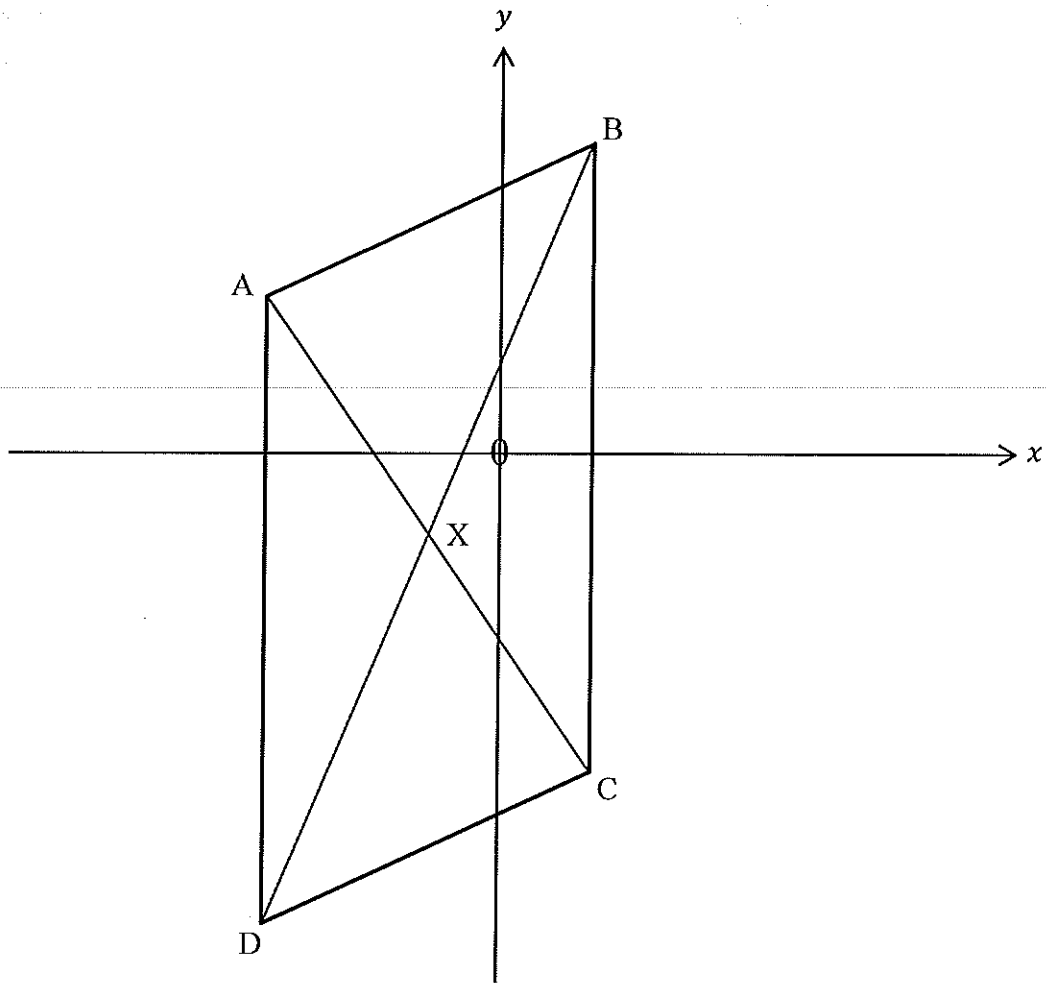
3. As people left the auditorium of a show, they were counted and timed.
The data was grouped and summarized as follows :

Time taken by people to leave the auditorium (minutes)	Number of people	Cumulative frequency
$3 < x \leq 6$	15	
$6 < x \leq 9$	25	
$9 < x \leq 12$	45	
$12 < x \leq 15$	20	
$15 < x \leq 18$	5	

- 3.1. Complete the table. (1)
- 3.2. Draw an ogive curve for the data. (3)
- 3.3. Estimate the number of people who left the auditorium after more than 7 minutes.
Clearly indicate, on your graph, where any values were read off or used and what they were. (2)
- 3.4.1. Write down the position of the upper quartile person. 1
- 3.4.2. Estimate how long the upper quartile person took to leave the auditorium.
Clearly indicate, on your graph, where any values were read off or used and what they were. 1 (2)
- 3.5. Estimate the standard deviation in the time taken for people to leave the auditorium. (2)

QUESTION 4 [22 marks]

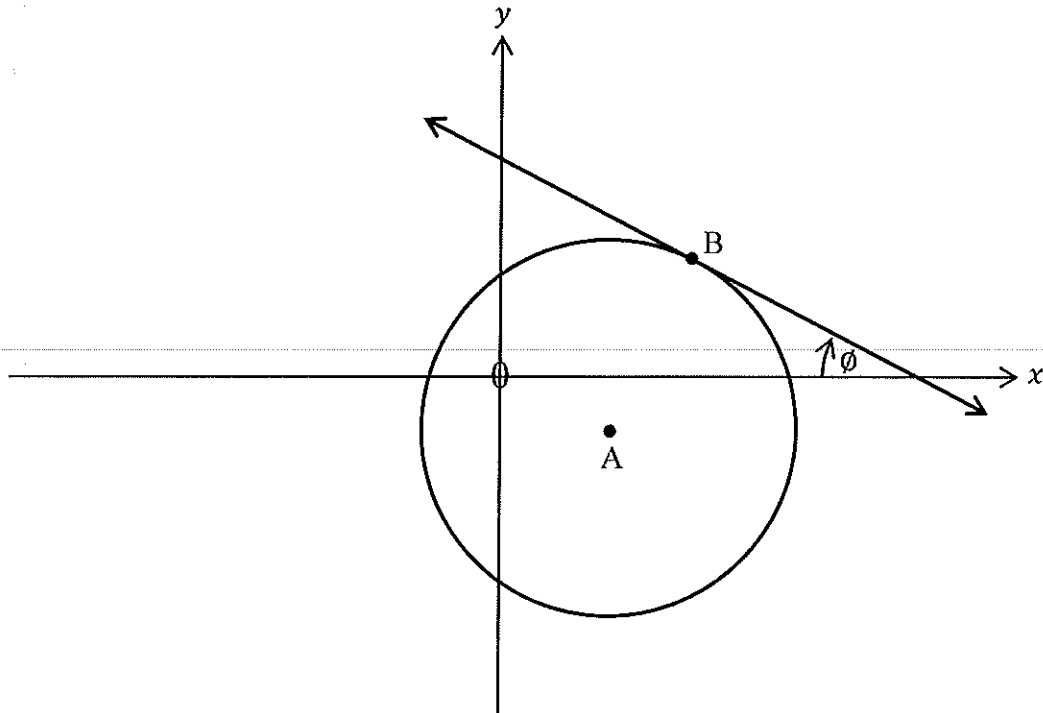
4. ABCD is a parallelogram. $A(-3; 2)$, $B(1; 4)$, $D(-3; e)$ and $X(f; -1)$:



- 4.1. Calculate the values of
- 4.1.1. f 1
- 4.1.2. e 2 (3)
- 4.2. Determine the equations of the lines
- 4.2.1. \overline{AD} 1
- 4.2.2. \overline{CD} 4 (5)
- 4.3. If A, B and $P(k; 10)$ are collinear, calculate the value(s) of k . (3)
- 4.4. Is $AX \perp XB$? Justify your answer. (5)
- 4.5. If $Q(y; -1)$ and $AQ = AB$, calculate the value(s) of y . (6)

QUESTION 5 [16 marks]

5. The equation of the circle, with centre A, is $x^2 - 4x + y^2 + 2y - 5 = 0$.
The straight line is tangential to the circle at point B and $\phi = 18,43494882^\circ$.



Determine the

- | | | |
|------|---|-----|
| 5.1. | coordinates of A, showing that they will be $(2; -1)$. | (3) |
| 5.2. | area of the circle. | (2) |
| 5.3. | gradient of the tangent, as a common fraction. | (3) |
| 5.4. | coordinates of B. | (8) |

QUESTION 6 [13 marks]

CALCULATORS MAY NOT BE USED IN THIS QUESTION

6.1. Use the identity : $\cos(x - y) = \cos x \cos y + \sin x \sin y$
and prove that : $\sin(x - y) = \sin x \cos y - \cos x \sin y$ (3)

6.2. If $\tan 10^\circ = k$, where $k > 0$, involve a diagram and determine $\cos 5^\circ$ in terms of k . Your answer does not need to be simplified. (5)

6.3. Simplify fully : $\frac{\cos(-234^\circ)}{(1 - 2\sin^2 15^\circ) \sin 18^\circ \cos 18^\circ}$ (5)

QUESTION 7 [16 marks]

7.1. Prove the identity : $\frac{1 + \tan \theta}{1 - \tan \theta} = \frac{1 + \sin 2\theta}{\cos 2\theta}$ (8)

7.2. Solve for x :

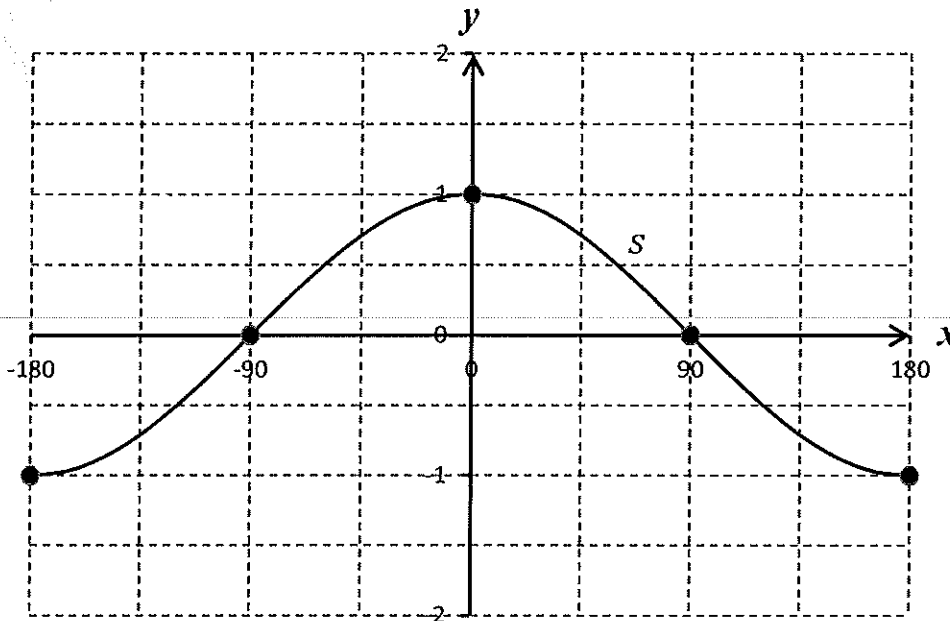
7.2.1. $\sin(3x - 10^\circ) + \cos 2x = 0$ 5

7.2.2. $\sin 2x + 3 \cos 2x = 0$ 3 (8)

QUESTION 8 [11 marks]

USE THE ANSWER SHEET PROVIDED

8.1. The graph of $s(x) = -\sin(x + m)$ is shown below :



Write down the value of m . (1)

8.2.1. On the same set of axes, sketch the graphs of :

$$f(x) = \tan x - 1 \quad \text{and} \quad g(x) = \cos 2x$$

for $x \in [-180^\circ; 180^\circ]$. 6

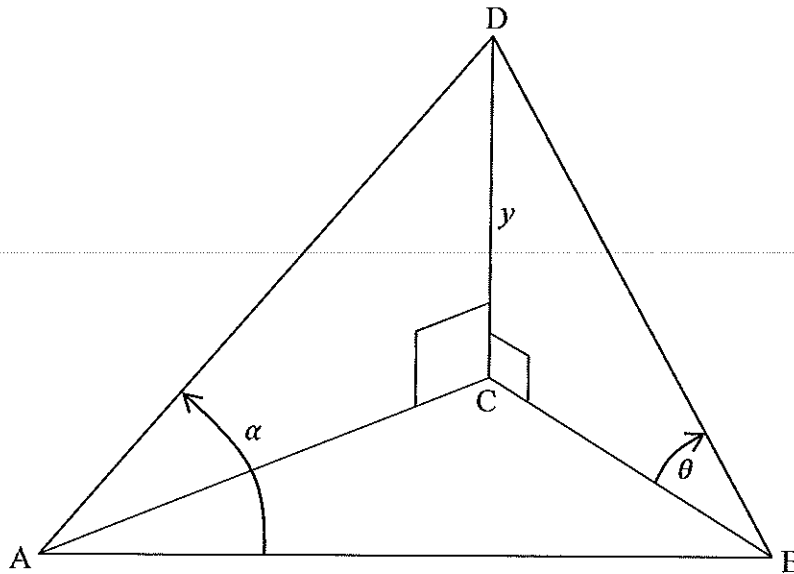
8.2.2. Use your graphs to solve for x , if $x \in [0^\circ; 180^\circ]$:

$$\cos 2x \tan x - \cos 2x \geq 0 \quad 4 \quad (10)$$

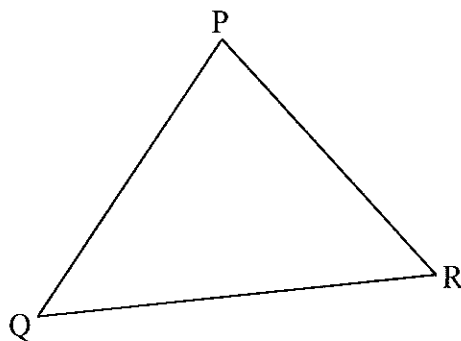
QUESTION 9 [10 marks]

USE THE ANSWER SHEET PROVIDED

- 9.1 A, B and C are points in the same horizontal plane. D is vertically above C and $DC = y$ metres in length. The angle of elevation D from B is θ .
 $\widehat{DAB} = \alpha$ and $DA = DB$.



- 9.1.1. Determine DB in terms of y and θ . 2
- 9.1.2. Hence, show that : $AB = \frac{2y \cos \alpha}{\sin \theta}$. 5 (7)
- 9.2. In the following diagram :



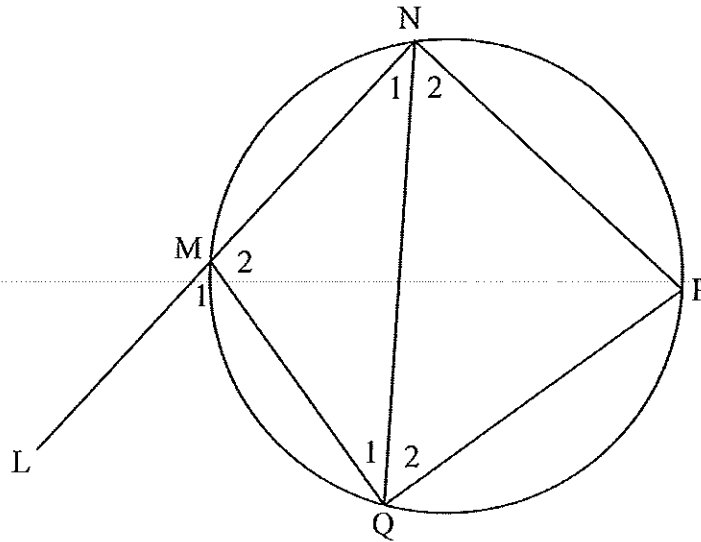
Prove that : $\cos \widehat{Q} = \frac{p - q \cos \widehat{R}}{r}$

HINT : Construct $PX \perp QR$ (3)

QUESTION 10 [23 marks]

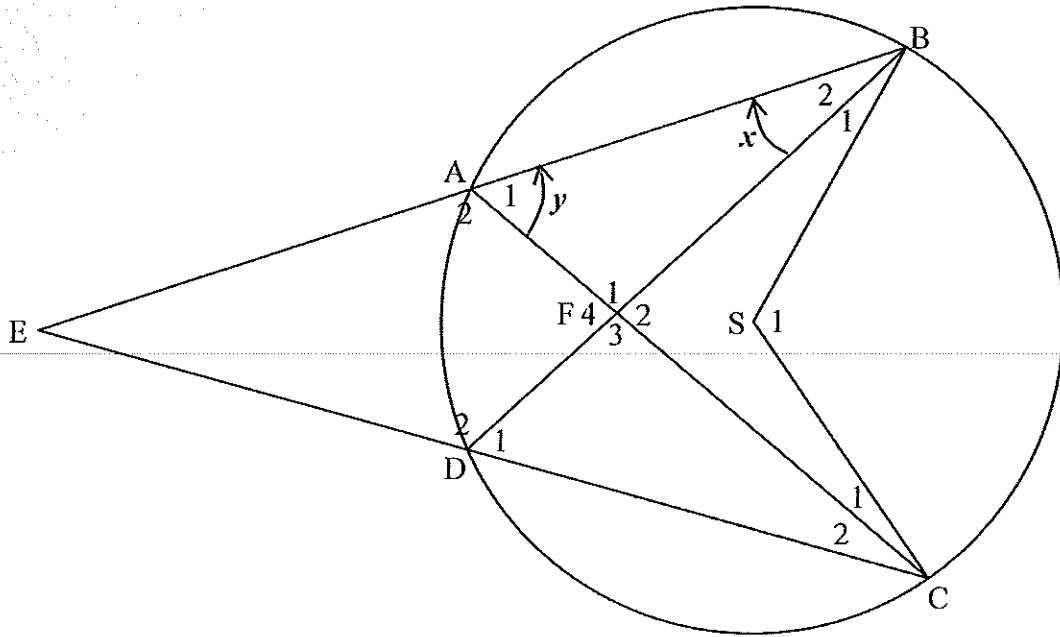
USE THE ANSWER SHEET PROVIDED

10.1. In the diagram, $\widehat{N}_1 = 35^\circ$, $\widehat{N}_2 = 45^\circ$ and $\widehat{Q}_1 = 50^\circ$.



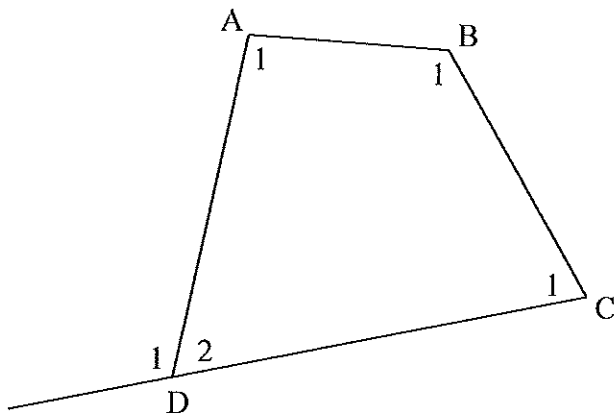
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|-----------|------------------------------|-----------------------|
| 10.1.1 | Determine \widehat{M}_1 . | <u>2</u> |
| 10.1.2.1. | Determine \widehat{Q}_2 | <u>3</u> |
| 10.1.2.2. | Hence, state why $MN = NP$. | <u>1</u> <u>4</u> (6) |

- 10.2. In the diagram, A, B and C are points on a circle whose centre is S. Chords BA and CD are produced to meet at E. AC and BD intersect at F. SB and SC are drawn. Let : $\widehat{ABD} = x$ and $\widehat{BAC} = y$.



- 10.2.1. Express \widehat{DFA} in terms of x and/or y . 1
 10.2.2. Prove that : $\widehat{BSC} = \widehat{DFA} + \widehat{E}$. 6 (7)

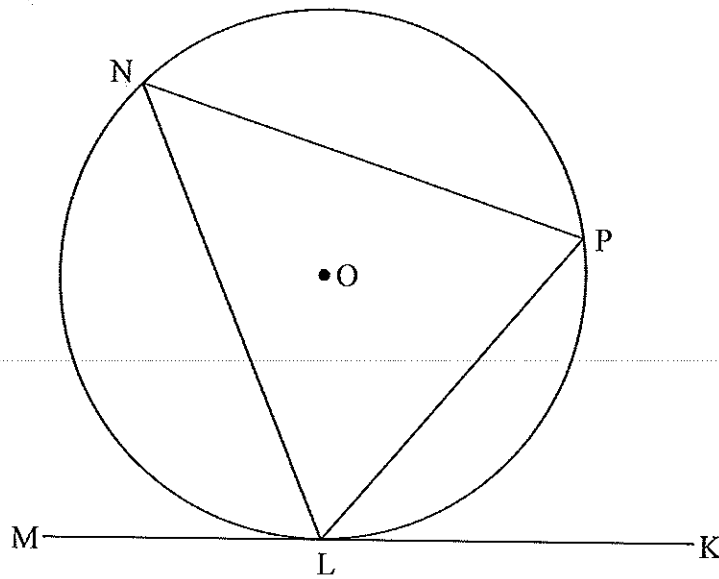
- 10.3. In the diagram, $\widehat{D}_1 = 6x + 30^\circ$, $\widehat{A}_1 = 5x - 10^\circ$, $\widehat{B}_1 = 8x - 10^\circ$ and $\widehat{C}_1 = 4x + 10^\circ$.



- 10.3.1. Prove that ABCD is a cyclic quadrilateral. 8
 10.3.2. Make a geometric observation about BD? Justify your observation. 2 (10)

QUESTION 11 [17 marks]

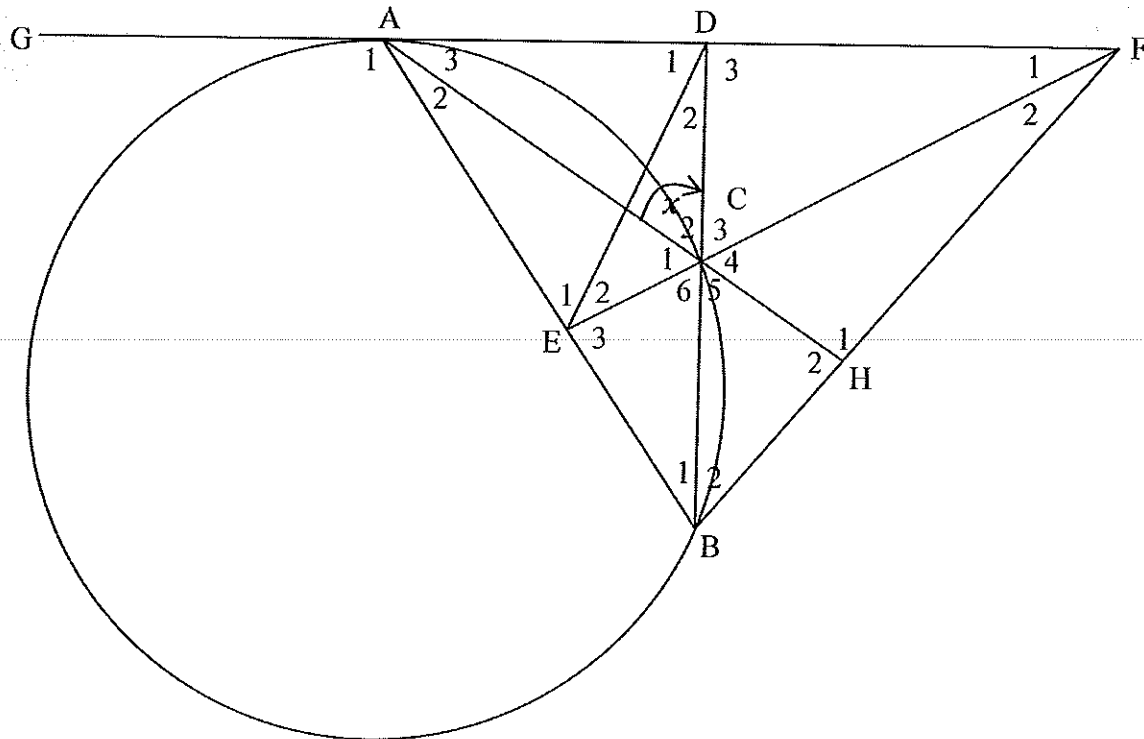
11.1. In the diagram, KM is a tangent to the circle, with centre O, at point L.



Prove the theorem which states that $\widehat{KLP} = \widehat{PNL}$.

(7)

- 11.2. In the diagram, GF is a tangent to the circle at A. AB is a chord, $BD \perp AF$ and BD intersects the circle at C. E is a point on AB such that $DE = DA$. BF is joined but is not a tangent to the circle. AC is produced to meet BF at H. Let : $\widehat{DCA} = x$.



Prove that :

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|---------|---|---------------|
| 11.2.1. | $\widehat{DCA} = \widehat{BAD}$ | <u>4</u> |
| 11.2.2. | ADCE is a cyclic quadrilateral | <u>4</u> |
| 11.2.3. | CD is the bisector of \widehat{ACF} . | <u>2</u> (10) |

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; \quad r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; \quad -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{area } \triangle ABC = \frac{1}{2} ab \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2\sin \alpha \cos \alpha$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

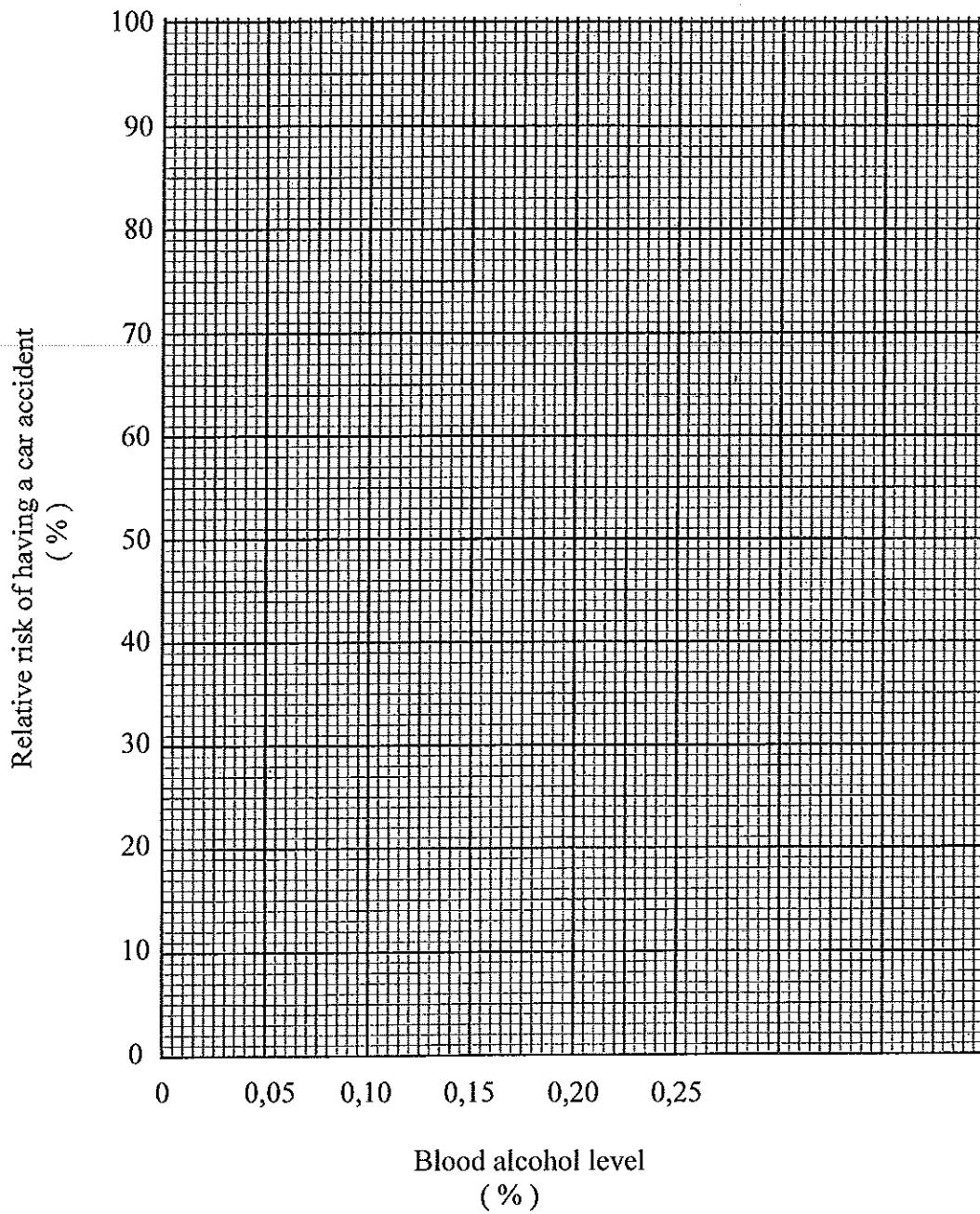
$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$

ANSWER SHEET FOR QUESTION 2

2.1.

Scatter plot of Relative risk of having a car accident versus Blood alcohol level



2.2. See diagram above.

2.3.
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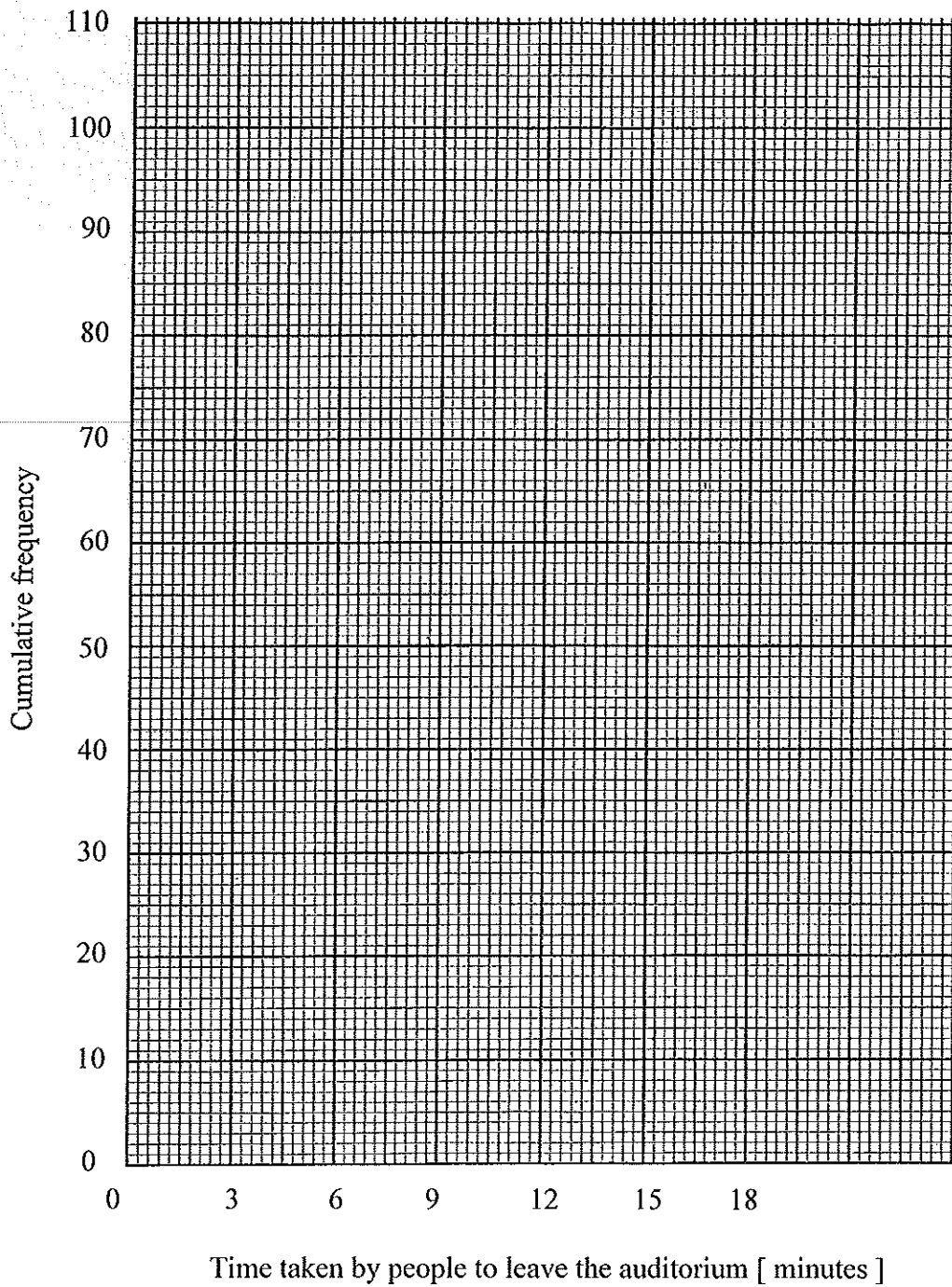
ANSWER SHEET FOR QUESTION 3

3.1.

Time taken by people to leave the auditorium (minutes)	Number of people	Cumulative frequency
$3 < x \leq 6$	15	
$6 < x \leq 9$	25	
$9 < x \leq 12$	45	
$12 < x \leq 15$	20	
$15 < x \leq 18$	5	

3.2.

Ogive curve of Time taken by people to leave an auditorium



3.3.

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3.4.1.

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3.4.2.

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3.5.

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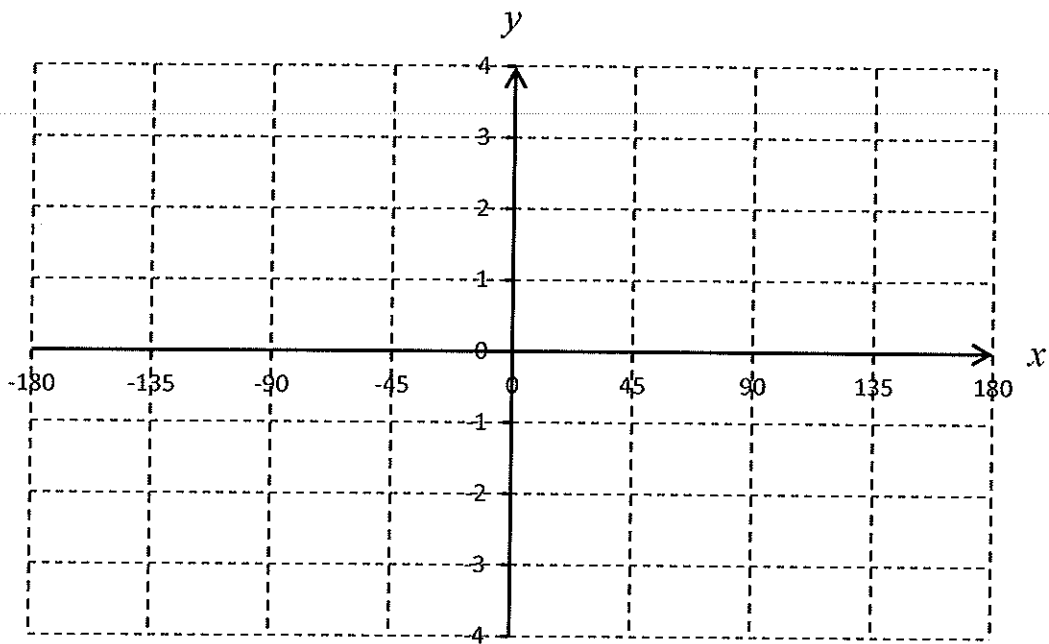
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ANSWER SHEET FOR QUESTION 8

8.1.
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8.2.1. $f(x) = \tan x - 1$ and $g(x) = \cos 2x$



8.2.2.
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